Calculus Resource

Information Regarding This Resource The following resource will help to unify the concepts of Limits, Continuity, Derivatives and Integrals by allowing students the opportunity to analyze data in a real-world scenario. Students will also collaborate, create and reflect on their learning.

The Format of This Resource This resource is organized into flexible components that can be utilized by educators, parents or students in its entirety or can be fragmented based on desired knowledge. Each text box contains the process skills students will use in the lesson to explore the mathematical content within the Calculus Curriculum. The focus of each lesson is highlighted for easy reference. The lessons have been designed to allow multiple entry points to accommodate for different levels of understanding. All lessons have been designed for mathematical computation and graphing without requiring a calculator.

Throughout this resource, students are asked to justify or explain their answers, thought process or understanding. The intent is for students to reflect on their mathematical thoughts. Students should keep in mind that justifications or explanations can take multiple forms, including, but not limited to, diagrams, graphs, text, or pictures. These are not meant to be right or wrong, rather a means of making learning visible.

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Connection of Standards:

Process Standard(s): Students will show their understanding of **limits and continuity** by making sense of problems, persevering, reasoning and making sense of relationships, using critical thinking skills to justify their mathematical reasoning, and connecting ideas to real world situations through modeling.

Content Standard(s):

C.LC.1 Understand the concept of a limit graphically, numerically, analytically, and contextually.

C.LC.2 Understand the definition and graphical interpretation of continuity of a function.

Sandy builds a chicken coop in her backyard and purchases chicks at the local feed store. When purchasing the chicks she is given instructions on how to care for them and she realizes that she will need a heat lamp, as the temperature must be closely monitored for the first 4 weeks. The instructions give the following constraints for temperature:

Temperature Requirements for Chicks in $°F$
95° the first week
90° the second week
85° the third week
75° the fourth week

Sandy wants a visual representation of this information so she can remember it more easily.

a. **Create a Temperature ($°F$) vs Time (weeks) graph of the above information.**
b. **Is this graph continuous from 0 weeks up to 5 weeks? Justify your answer referring to your visual representation.**
c. **Find the following limits:**
   $\lim_{w\to 1} T(w)$ $\lim_{w\to 2} T(w)$ $\lim_{w\to 3} T(w)$ $\lim_{w\to 4} T(w)$
d. **Do all of the above limits exist? What do the limits mean in terms of the chick's age and temperature?**
e. **Describe another real-world situation in which it is beneficial to understand continuity and limits.**

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Connection of Standards:

Process Standard(s): Students will show their understanding of **differentiation** by making sense of problems, persevering, reasoning and making sense of relationships, using critical thinking skills to justify their mathematical reasoning, and connecting ideas to real world situations through modeling.

Content Standard(s):

C.D.1 Understand the concept of the derivative of a function geometrically, numerically, analytically, and verbally.

C.D.2 Apply the rules of differentiation to functions.

You receive a package in the mail as a promotion from a new airline that is now offering flights out of an airport near you. The package contains a small rocket with the airline's logo written on the side. You run outside to get it set up for launch! Below is the advertisement enclosed in the box. This seems unbelievable!!

Blast Off!!!
Enjoy the experience of flight with vertical distances surpassing 70 feet and velocities of over 30mph!!!

While setting up the rocket, you notice an equation in small print under the instructions that predicts the path of the rocket, $H(t) = -16t^2 + 64t$. You remember that in your Calculus class you learned how to determine the velocity and acceleration of an object when given the distance vs time graph. Can this equation help to determine if the brochure contains accurate information?

a. **Create a Distance (feet) vs Time (seconds) graph using the above function.**
b. **What information can you derive regarding the rocket's path from analyzing the graph?**
c. **Does this information help in determining the accuracy of the company's advertisement? Justify your answer.**
d. **Take the first derivative of the above function. What information does this derivative give regarding the rocket's path?**
e. **Graph the first derivative to justify your answer.**
f. **Take the second derivative of the function. What information does this give regarding the rocket? Justify the second derivative in terms of the rocket.**
g. **Justify the validity of the advertisement. Pay close attention to units and use mathematical reasoning in your justification.**
h. **What other real-world phenomena rely on derivatives for analysis?**

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Connection of Standards:

Process Standard(s): Students will show their understanding of **the rules of differentiation** by making sense of problems, persevering, reasoning and making sense of relationships, and using critical thinking skills to justify their mathematical reasoning.

Content Standard(s):

C.D.3 Apply theorems and rules of differentiation to solve mathematical and real-world problems.

Rules of Differentiation:

Sum Rule: $\frac{d}{dx}f(x) + \frac{d}{dx}g(x) = \frac{d}{dx}[f(x) + g(x)]$

Difference Rule: $\frac{d}{dx}f(x) - \frac{d}{dx}g(x) = \frac{d}{dx}[f(x) - g(x)]$

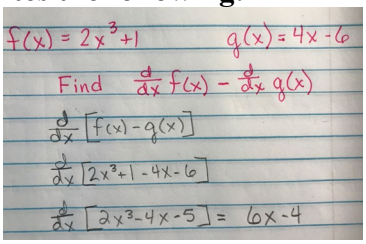
Constant Multiple Rule: $\frac{d}{dx}[k \cdot f(x)] = k \cdot \frac{d}{dx}f(x)$

Constant Rule: $\frac{d}{dx}k = 0$

Your instructor has become ill and has asked you to step in to explain the Rules of Differentiation to an Honors Calculus class.

a. **Apply each of the rules using mathematical examples you have created.**
b. **Justify your mathematical application using explicit language.**

You then ask a student to use the Difference Rule to solve a problem you have written on the board. The student writes the following:



c. **You determine that the student's final answer is incorrect because he did not subtract the functions correctly within grouping. Mathematically justify your reasoning to determine if you gave the student accurate information.**

You have done a great job in your explanation and mathematical reasoning. Now a student asks when anyone would ever use this in real life. You tell the student that you will get back to her tomorrow.

d. **Research different careers that use calculus differentiation in their work. Explain at least two different career opportunities that use differentiation, on a regular basis, in their line of work.**

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Connection of Standards:

Process Standard(s): Students will show their understanding of **the integral of a function** by making sense of problems, persevering, reasoning and making sense of relationships, using critical thinking skills to justify their mathematical reasoning, and connecting ideas to real world situations through modeling.

Content Standard(s):

C.I.1 Understand the concept of the integral of a function geometrically, numerically, analytically, and contextually.

The rate of change of a population in a Midwestern city is modeled by the exponential function $P'(t) = 12.48e^{0.026t}$, where t is the number of years since 1958 and $P'(t)$ is the rate in thousands of people per year. In 1976, the population was 796,000.

a. **Find the population model $P(t)$. Justify your model mathematically using integration.**
b. **Use this model to estimate what the population will be in 2022.**
c. **Does this model seem reasonable? Justify your answer.**

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Connection of Standards:

Process Standard(s): Students will show their understanding of **the rules of integration** by persevering, reasoning and making sense of relationships, and using critical thinking skills to justify their mathematical reasoning.

Content Standard(s):

C.I.2 Apply theorems and rules of integration to solve mathematical and real-world problems.

The Fundamental Theorem of Calculus is comprised of the First and Second Theorems of Calculus. The First Theorem of Calculus relates to differentiation and the Second Theorem of Calculus relates to integration.

a. **Your friend has a Calculus test tomorrow over integrals! He asks you what integration means and how it is related to differentiation. Write an explanation to your friend, in your own words, your meaning of integration and how differentiation and integration are related. Be specific. Your friend has a test tomorrow, so you are explaining for understanding.**
b. **Think about your understanding of integration. If you sat down face to face with your teacher and asked any two questions for clarification on integration or differentiation, what would the questions be? Think about what you still don't understand about differentiation and integration.**
c. **Use the internet, a teacher, a friend, or text to research your two questions. Answer your questions in detail to confirm you now have a better understanding.**
d. **Give an example of two careers that incorporate integration within their work scope. Be specific in your explanation as to how and why it is used, and what the final product accomplishes.**

Reflection:

a. **Collaborate with someone in your family, a friend, or a neighbor. Ask them to look over your mathematical reasoning and ask you at least 5 guiding questions. Document the 5 questions they asked.**
b. **Document the answers you gave to the 5 questions.**
c. **Reflect on your work. Where did you struggle? Where did you triumph? What do you still wonder?**
d. **Look over the content you covered, the processes that guided you through your discoveries, and think about your collaboration. Write down your thoughts and allow your reflection to move you forward in your mathematical thinking.**