Note to the Educator and Caregiver. This Math Task addresses the following Content Standards and is separated into different scaffolded math tasks that build on top of one another and should take place over several days. The Process Standards that are incorporated throughout the lessons are listed below the contents standards. Following the process standards is an overview of the lesson for students and parents. There is an answer document at the end of this task for students to record their findings.

FA. FIF.1 Extend previous knowledge of a function to apply to general behavior and features of a function.

a. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.
b. Represent a function using function notation and explain that (x) denotes the output of function f that corresponds to the input x.
c. Understand that the graph of a function labeled as f is the set of all ordered pairs $(x,)$ that satisfy the equation $y=(x)$.

FA.FIF.2 Evaluate functions and interpret the meaning of expressions involving function notation from a mathematical perspective and in terms of the context when the function describes a real-world situation.
FA.FIF.4* Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features. Key features include intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity. (Limit to linear; quadratic; exponential.)
FA.FIF.5* Relate the domain and range of a function to its graph and, where applicable, to the quantitative relationship it describes. (Limit to linear; quadratic; exponential.)
FA.FIF.7 Graph functions from their symbolic representations. Indicate key features including intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity. Graph simple cases by hand and use technology for complicated cases. (Limit to linear; quadratic; exponential only in the form $y=ax+k$.)
FA.FIF.9* Compare properties of two functions given in different representations such as algebraic, graphical, tabular, or verbal. (Limit to linear; quadratic; exponential.)
FA.FLQE.1* Distinguish between situations that can be modelled with linear functions or exponential functions by recognizing situations in which one quantity changes at a constant rate per unit interval as opposed to those in which a quantity changes by a constant percent rate per unit interval.

a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.

FA.FLQE.3* Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or more generally as a polynomial function.
FA.FLQE.5* Interpret the parameters in a linear or exponential function in terms of the context. (Limit to linear.)
FA.FBF.3 Describe the effect of the transformations (x), $(x)+k$, $f(x+k)$, and combinations of such transformations on the graph of $y=f(x)$ for any real number k. Find the value of k given the graphs and write the equation of a transformed parent function given its graph. (Limit to linear; quadratic; exponential with integer exponents; vertical shift and vertical stretch.)

Process Standards:

1. Make sense of problems and persevere in solving them.
    a. Relate a problem to prior knowledge.
    b. Recognize there may be multiple entry points to a problem and more than one path to a solution.
    d. Evaluate the success of an approach to solve a problem and refine it if necessary.
2. Reason both contextually and abstractly.
    a. Make sense of quantities and their relationships in mathematical and real-world situations.
    b. Describe a given situation using multiple mathematical representations.
    c. Translate among multiple mathematical representations and compare the meanings each representation conveys about the situation.
3. Use critical thinking skills to justify mathematical reasoning and critique the reasoning of others.
    c. Make conjectures and explore their validity.
    d. Reflect on and provide thoughtful responses to the reasoning of others.
4. Connect mathematical ideas and real-world situations through modelling.
    a. Identify relevant quantities and develop a model to describe their relationships.
    b. Interpret mathematical models in the context of the situation.
6. Communicate mathematically and approach mathematical situations with precision.
    b. Represent numbers in an appropriate form according to the context of the situation.
    c. Use appropriate and precise mathematical language.
    d. Use appropriate units, scales, and labels.
7. Identify and utilize structure and patterns.
    b. Recognize mathematical repetition in order to make generalizations.

About the Lesson:
Graphical models surround us every day, especially with the current daily models that show the spread of the Coronavirus. Information or data can be relayed in different ways, including written articles, audio, video or pictorials. A graph is an example of a pictorial model that allow us to quickly see what is happening with data. For example, we can easily see trends that are increasing or decreasing, how quickly they are increasing or decreasing and how they are increasing or decreasing, at a constant rate, exponentially, randomly, etc… When we look at data tables or graphs of Coronavirus statistics, we can see that the rate is not increasing at a constant rate, but very rapidly. We might say the cases of Coronavirus are increasing exponentially, or with a rapidly increasing rate of change, but what does that really mean?

The following pages will allow you to investigate graphical information. You will investigate how to analyze the information in a table of values and then in its pictorial graph. You will also look at how data increases or decreases and under what circumstances the different models might exist.

This lesson consists of four math tasks. The first task is designed for you to process information and confirm your understanding. It is a story about Sahara and her walk through her neighborhood. This story walks you through Sahara's learning and her graphical analysis to make her learning visible to you. There is a brief introduction at the beginning of the story to successfully guide you through reading for understanding. In the second task "Tell Your Story", you will apply what you learned from Sahara by creating your own story using a graphical model. The third task is about Frank saving money. He loves video games and wants to buy a new computer, but needs assistance in determining the best savings plan. In the fourth and last task, you have been promoted at work and now supervise three employees. You have been assigned the task of writing an evaluative report for the three employees based off a productivity graph.

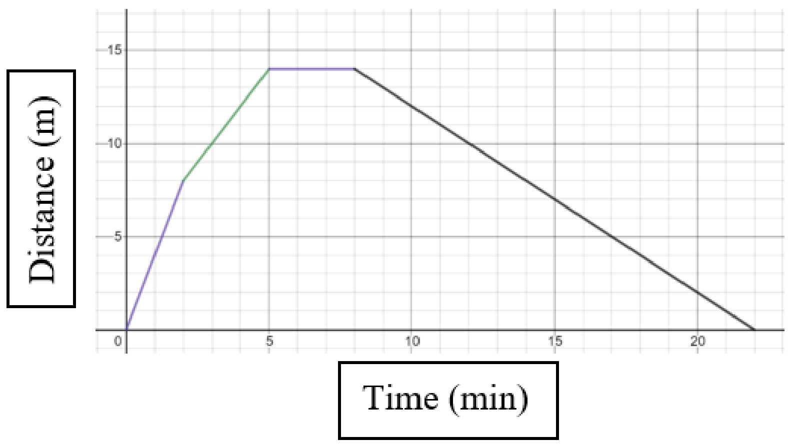
Let's get started!!!

"A Walk Through the Neighborhood" is a story meant to read as a think aloud model of graphical interpretation and unit conversion. The story has been broken into sections to allow processing of each concept before moving on to new ideas. Some of the concepts might require a second or third read before understanding occurs, so don't get overwhelmed if you don't understand the first time. Be patient and take your time. Allow yourself room for error and the opportunity to work at your own pace. It is not a race for completion, but a task for understanding.

A Walk Through the Neighborhood

Sahara decides to walk around her neighborhood for exercise. She is wearing a new device that will calculate how far she walks over a period of time and will graph the data. The device records her distance in meters and her time in minutes. Sahara cannot figure out how to change the setting so that her distance is recorded in miles and her time in hours, as miles/hour makes much more sense to her. She begins walking and decides she will worry about the conversion after she gets home.

Below is the graph that Sahara's device created after she returned home from her walk.



Sahara is determined to figure out how far and how quickly she walked so she sits at her desk and begins analyzing the graph from her new device. She records the following information on a piece of paper:

- The first 2 minutes I walked 8 meters.
- After 5 minutes I walked a total of 14 meters.
- After 8 minutes I was still at 14 meters. (This must have been when I stopped to tie my shoe!)
- After my shoe was tied, I turned around and started walking home.
- I returned home after 22 minutes.

Stop at this point and look carefully at the graph while reading Sahara's notes. Make sure that you understand the relationship between the graph and her explanation.

If you can answer the question, then answer it and move on. If you are struggling to answer the question, reread the analysis and then try the question again.

1. **Do the notes that Sahara wrote on her paper match the graphical model? Justify your answer by explaining the four line segments that Sahara analyzed.**

Sahara now decides to analyze the graph one piece at a time to figure out how fast she walked. She thinks that she walked about the same speed the first 2 minutes, because the purple line on the graph is straight and has the same slope. She decides to calculate this speed first!

She knows she walked 8 meters in 2 minutes. Hmmmmm… Sahara remembers something else she learned in her math class, that rate of change is the same as slope. This is all starting to make sense now! Since slope is $\frac{rise}{run}$, which means $\frac{\Delta y}{\Delta x}$, and the y-values are meters and the x-values are minutes, then the slope written as a rate, would be $\frac{8\ meters}{2\ minutes}$. My goodness, she is finally starting to understand! Now to keep going….

Since 8 is divisible by 2, that would mean that she walked 4 meters in 1 second! So, her rate was $4\frac{meters}{minute}$!

Stop at this point and look over the text. Make sure you understand the connection between the slope of the line and the rate of change.

If you can answer the question, then answer it and move on. If you are struggling to answer the question, reread the analysis and then try the question again.

2. **Use the graphical model to justify that Sahara's speed was $4\frac{meters}{minute}$ for the first part of her journey. Use your own words and what you see in the graph that leads to this conclusion.**

Now for the green line! Sahara recognizes that the green line has a different slope and looks like it's not quite as steep as the purple line, which leads her to believe that she walked slower from 2 to 5 minutes. She determines the time she walked this slower speed was 3 minutes because that is the difference between 2 and 5 minutes. She also concludes that over the 3 minutes, she walked 3 meters because that is the difference between 8 meters and 14 meters. This means that she traveled 6 meters in 3 minutes, or $\frac{6\ meters}{3\ minutes}$ $= 2\frac{meters}{minute}$!

Whew! This is tough, but I think we're getting somewhere! Now for the rest of the graph!

After the 8 minutes when Sahara stopped to tie her shoe, the graph shows a negative slope. This confuses Sahara! She thinks this might mean that she was slowing down. But that doesn't make sense to her. Then she looks back at the graph and thinks about what the black line is really modeling. Since the y-axis is distance from 0 and her house is where she started, or 0, then the black line must be showing that her distance is decreasing until it finally reaches 0, or her home. This makes much more sense! So Sahara decides that for the last part of her journey she walked 14 meters in 14 minutes in the direction of her house. So, $\frac{14\ meters}{14\ minutes} = 1\frac{meter}{minute}$.

Stop at this point and look over the text. Make sure you understand the connection between a negative slope and Sahara's return home.

If you can answer the question, then answer it and move on. If you are struggling to answer the question, reread the analysis and then try the question again.

3. **Sahara first thought that the negative slope meant that her speed was slowing, but then corrected herself and determined it was when she was walking back home. Explain why, in terms of distance and time, the negative slope means that Sahara was returning home and not slowing down.**

Eureka!!! Not too fast though! There is another problem! Sahara mentioned that she doesn't really understand what $\frac{meters}{minute}$ is telling her about her speed. She really understands $\frac{miles}{hour}$ because she can relate that to the speed of her mother's car. Now to figure out how fast each of these speeds are in $\frac{miles}{hour}$.

Sahara decides to start with the purple line, but she's a little stumped on which unit to convert first, meters or minutes… She's not sure where to start, but decides she has to start somewhere, so she decides to convert the meters to miles first. She remembers that any equivalent units are written within one ratio, for example $\frac{1\ hour}{60\ minutes}$ or $\frac{60\ minutes}{1\ hours}$. Also, any like units cancel into 1 if one is located in the numerator and the other in the denominator, for example $\frac{10\ ft \cdot sec}{sec} = 10$ ft., because $\frac{sec}{sec} = 1$.

She also remembers some conversions that her teacher taught her!!! She remembers that there are 2.54 cm in 1 inch and 5,280 feet in 1 mile. These should help!

She writes on her paper: $\frac{4\ \cancel{meters}}{1\ minute}\left(\frac{100\ \cancel{cm}}{1\ \cancel{m}}\right)\left(\frac{1\ \cancel{inch}}{2.54\ \cancel{cm}}\right)\left(\frac{1\ \cancel{ft}}{12\ \cancel{in}}\right)\left(\frac{1\ mile}{5280\ \cancel{ft}}\right) = \frac{400\ miles}{160{,}934.4\ minutes} = .0025\frac{miles}{minutes}$

Now she decides to tackle the minutes to convert it to hours!

$$.0025\frac{miles}{\cancel{minutes}}\left(\frac{60\ \cancel{min}}{1\ hour}\right) = 0.15\frac{miles}{hour}$$

Stop at this point and look over the unit conversion.

If you can answer the question, then answer it and move on. If you are struggling to answer the question, reread the analysis and then try the question again.

4. **Explain why Sahara placed the units where she did in each ratio of the conversion. In other words, why did she write $\frac{cm}{m}$ in the first ratio and not $\frac{m}{cm}$.**

Now it should be quicker for her to move through the other rate of change from 2 minutes to 5 minutes where she advanced from 8 meters to 14 meters, which means she traveled 6 meters in 3 minutes.

$$\frac{6\ meters}{3\ minutes} = 2\frac{\cancel{meters}}{\cancel{minute}}\left(\frac{100\ \cancel{cm}}{1\ \cancel{meter}}\right)\left(\frac{1\ \cancel{inch}}{2.54\ \cancel{cm}}\right)\left(\frac{1\ \cancel{ft}}{12\ \cancel{in}}\right)\left(\frac{1\ mile}{5280\ \cancel{ft}}\right)\left(\frac{60\ \cancel{min}}{1\ hour}\right) = \frac{12{,}000\ miles}{160{,}934.40\ hours} = .075\frac{mi}{hr}$$

Because there is a zero slope, or no distance walked from 5 – 8 minutes, the speed, or rate of change, was 0.

She continues to use her unit conversions and calculates the speed while she was walking home to be $.04\frac{miles}{hour}$.

You've made it!! Take a shot at these last questions. You know the procedure! If you can answer them successfully answer them and move on, if not reread the above to clarify!

5. **Does this speed, for the last segment, seem reasonable? Without doing any calculation, explain why $.04\frac{miles}{hour}$ does or does not seem logical.**
6. **Now calculate the speed and see if you were correct in your prediction!**

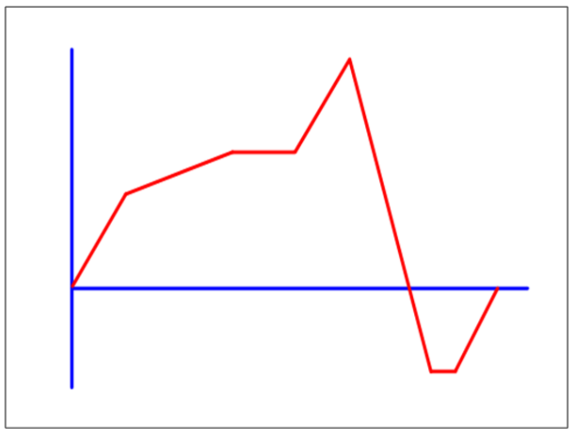
Tell Your Story!!!

The story that you create will be based around the below graph. The x- and y-axes have not been labeled to prevent limitations, allowing you to be extra creative!!!

Start by thinking of a story that would be meaningful to you and then determine if the graphical representation could somehow represent this story. You might need to make a few adjustments as you think through your story to allow a connection between the graph and your thoughts.

Once you have identified a good plan, label the x- and y-axis and give numerical values to both. You will be asked to find at least two rates of change after you are finished with your story, so numbers will be important.

Now it's time to write your story!!! When you are finished, answer the questions below.



Questions:

1. **What does it mean, in terms of YOUR story, when the graph goes below the x-axis?**
2. **In most real-world examples, will the x-values be negative? Explain your reasoning.**
3. **Find the average rate of change over two different x-intervals. (Make sure to include units.)**
4. **Why is the rate of change important for the story you created? What information does it give?**

$ Saving Money $

Frank loves to play video games and wants to purchase a new computer. He has priced different computers and has found one that he likes for $1,356 including tax. Frank realizes that he has to start saving money if he wants to purchase the computer. He is considering two different ways to deposit money into his savings account. Frank is unsure which plan will allow him to save $1,356 quicker. To help Frank with his dilemma, complete the two tables below.

A. Frank deposits $100 into a savings account and plans to deposit $100 every month.

Time (month)	Dollars in Account
0	100
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

B. Frank deposits $0.50 into a savings account and plans to double the amount he deposits every month.

Time (month)	Dollars in Account
0	0.50
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

Questions:

1. **Explain which method of saving money would allow Frank to purchase the computer quicker.**
2. **Graph both tables of values. (Graph paper is provided at the end of this document if needed.)When graphing, make sure to include even increments and labels for both axes.**
3. **Now that you have a table of values and a graphical model for each of the above scenarios, use the below definitions of linear, quadratic, and exponential functions to determine the type of function each scenario represents.**

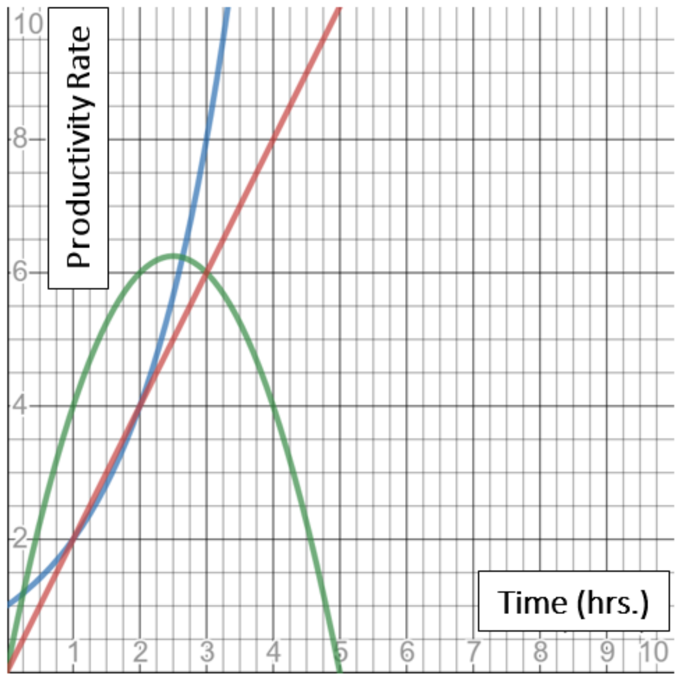
> **<u>Linear Functions</u> are algebraic equations whose graphs form straight lines.**
> **<u>Quadratic Functions</u> are equations whose graphs form parabolas, which are U-shaped graphs facing upward or downward.**
> **<u>Exponential Functions</u> are equations whose graphs increase or decrease rapidly.**

4. **Explain, in detail, how you know which type of function each table and/or graph represents.**
5. **Explain each graph separately in terms of rate of change. Does this help in the identification of the function type?**

You're the Boss!!!

You just received a promotion at work!!! One of your responsibilities is to monitor the productivity rate (*the effectiveness of productive effort, or the rate of output per unit of input*) of 3 employees. Below, is a visual representation of the three employees productivity rate over a 5 hour period between 8:00 a.m. and 1:00 p.m. (The employees take lunch from 1:00 – 1:30.)

Your supervisor has asked you to compile a performance evaluation on each employee. To protect the employees identities, the company has color coded the graphs instead of using names. We will refer to the employees as Blue, Red and Green. The below questions will help you write the performance report.



(Time 0 on the graph corresponds to 8:00 a.m., when their workday starts)

Questions:

1. **Within the report, give an in-depth analysis of each employee separately. Here are a few things to think about when writing your evaluations. (Label each evaluation Blue, Red and Green)**
   - **When is productivity increasing and decreasing between 8:00 a.m. and 1:00p.m.?**
   - **When is productivity increasing the most rapidly between 8:00 a.m. and 1:00 p.m.?**
   - **When does productivity seem to be lowest between 8:00 a.m. and 1:00 p.m.?**
   - **Which employee has the best overall productivity? What do you see on the graph that leads to this assumption?**
   - **Which employee has the lowest overall productivity? What do you see on the graph that leads to this assumption?**
   - **What might it mean for each person to have a different productivity rate at 8:00 a.m.?**
2. **Write a conclusion for the performance report for all three employees.**
3. **Your supervisor sends you a memo which states that the company budget allows for one employee to get a substantial raise. He asks for your opinion on which of the three employees should receive that raise. Write a memo to your supervisor stating your opinion and justify your choice.**

GREAT JOB STUDENTS!! YOU ARE NOW MASTER DATA ANALYSTS!!!

e-Learning Math Task Answer Document

A Walk Through the Neighborhood

1.

2.

3.

4.

5. ____

6.

Tell Your Story!!!

Title: ______________________________

Questions 1-4:

1.

2.

3.

4.

$ Saving Money $

A. Frank deposits $100 into a savings account and plans to deposit $100 every month.

Time (month)	Dollars in Account
0	100
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

B. Frank deposits $0.50 into a savings account and plans to double the amount he deposits every month.

Time (month)	Dollars in Account
0	0.50
1	
2	
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5	
6	
7	
8	
9	
10	
11	
12	

1. ____

2. (You are not required to use both grids. You may choose to graph both data tables on one grid.)

3. ______________________________

4. ______________________________

5. ______________________________

You're the Boss!!!

Analysis For Each Employee:

1.

Conclusion:

2.

Memo:

3.