Intermediate Algebra

The following resource will help to unify the concepts of Expressions, Functions and Inequalities and the Complex Number System by allowing students the opportunity to analyze information and create personal representations that allow a better understanding of the concepts. Students will also collaborate, create and reflect on their learning.

The organization of this resource connects process standards with content, allowing students the opportunity to connect their own ideas as they explore the mathematical content within the Intermediate Algebra Curriculum. It is also designed to allow multiple entry points to accommodate for different levels of understanding. Teachers have the flexibility of using the document in entirety or using specific standards to reinforce prior knowledge.
(When students justify their answers, they can show their reasoning in any way that makes sense to them. There are no incorrect means of justification as long as the student's thought process is visible.)

Connection of Standards:

Process Standard(s):

Students will show their understanding of i**nterpreting information and modeling given a quadratic equation** by making sense of problems, persevering, reasoning and making sense of relationships, using critical thinking skills to justify their mathematical reasoning, and connecting ideas to real world situations through modeling.

Content Standard(s): IA.FIF.6* Given a function in graphical, symbolic, or tabular form, determine the average rate of change of the function over a specified interval. Interpret the meaning of the average rate of change in a given context.

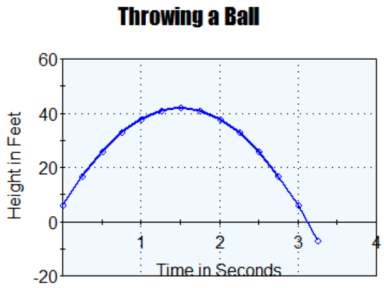
IA.FIF.7* Graph functions from their symbolic representations. Indicate key features including intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity. Graph simple cases by hand and use technology for complicated cases.

IA.ACE.2* Create equations in two or more variables to represent relationships between quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales.

A ball is thrown into the air. Refer to the below graph to answer the questions about the ball's path. It is represented by the equation $f(x) = -16t^2 + 48t + 6$, where t is time in seconds and $f(t)$ is distance in feet.

a. Why do you think that the leading coefficient is negative?
b. How does the leading coefficient effect the shape of the graph?
c. Looking at the graph below, approximately how long was the ball in the air? How could you check your answer to determine accuracy?
d. How high did the ball travel? How could you mathematically justify this answer?
e. Calculate the average rate of change over the interval [1,3]?
f. Is the rate of change positive or negative?
g. What does that positive or negative number mean in terms of rate of change?
h. Approximately, how high off the ground was the ball released?
i. Justify the height mathematically.

j. Explain the benefits of having a graph to accompany an equation.
k. What are the benefits of having the equation to accompany the graph? (Think in terms of approximations vs precise answers.)



Identify an experience that represents a quadratic function.

a. Graph the quadratic function that represents this experience
b. Write an equation that you feel best fits the graph.
c. What information or technology might have made this task easier to accomplish?

Connection of Standards:

Process Standard(s):

Students will show their understanding **of interpreting information given an exponential model** by making sense of problems, persevering, reasoning and making sense of relationships, using critical thinking skills to justify their mathematical reasoning, and connecting ideas to real world situations through modeling.

Content Standard(s):

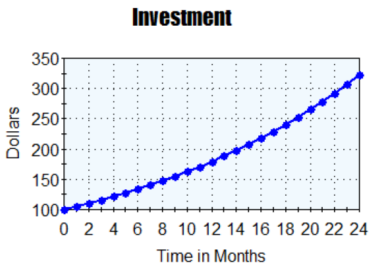
IA.FIF.6* Given a function in graphical, symbolic, or tabular form, determine the average rate of change of the function over a specified interval. Interpret the meaning of the average rate of change in a given context.

IA.FIF.7* Graph functions from their symbolic representations. Indicate key features including intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity. Graph simple cases by hand and use technology for complicated cases.

IA.FLQE.5* Interpret the parameters in a linear or exponential function in terms of the context.

The below graph shows an investment that draws 5% interest per month.

a. Does this graph represent exponential growth or decay?
b. Justify your answer to the above question. How do you know that it is growth or decay?
c. What is the y-intercept for this graph and what does that represent in terms of the investment?
d. After how much time should the investment yield at least $200?
e. Approximately, how much does the investment yield after 22 months?
f. After 3 years, how much do you predict the investment will be worth?
g. Is this investment realistic? Justify your answer to explain why you believe it is realistic or why it is not realistic.



Intermediate Algebra

Think of a real world example that might represent exponential *decay* instead of growth. Write an equation for the scenario identifying the initial value, decay rate and decay factor. Draw a visual representation of this example. Label both axes with units and labels. After you have drawn the graph, create 3 questions relating to the graph and answer the questions.

Connection of Standards:

Process Standard(s): Students will show their understanding of **solving radical equations** by making sense of problems, persevering, reasoning and making sense of relationships, and using critical thinking skills to justify their mathematical reasoning.

Content Standard(s):

IA.AREI.2* Solve simple rational and radical equations in one variable and understand how extraneous solutions may arise.

If $x^{\frac{p}{t}} = \sqrt[t]{x^p}$, then solve the equation $\left((x-2)^{\frac{2}{3}}\right) = \sqrt[3]{(2x+1)^2}$.

a. Justify and test your solution.
b. Does the solution indeed work for the equation or is it extraneous?

Connection of Standards:

Process Standard(s): Students will show their understanding of **analyzing polynomials in multiple forms** by making sense of problems, persevering, reasoning and making sense of relationships, and using critical thinking skills to justify their mathematical reasoning.

Content Standard(s):

IA.ASE.2* Analyze the structure of binomials, trinomials, and other polynomials in order to rewrite equivalent expressions.

IA.FIF.4* Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features. Key features include intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity.

IA.FIF.7* Graph functions from their symbolic representations. Indicate key features including intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity. Graph simple cases by hand and use technology for complicated cases. IA.FIF.8* Translate between different but equivalent forms of a function equation to reveal and explain different properties of the function.

a. Write the expression in an equivalent form: $(x-2)^2(x+1)^3$
b. How could you mathematically test to confirm these expressions are equivalent?
c. Classify the polynomial in terms of its degree and terms.
d. Can you classify it by degrees and number of terms in both of the equivalent forms?
e. Explain your reasoning.
f. Rewrite this polynomial expression as an equation. What would the graph of the equation look like? List as many key features as possible.

g. If you wanted to reflect the graph over the x-axis, what change would you make to the equation?

Connection of Standards:

Process Standard(s):

Students will show their understanding of **using real-world information to write and solve equations** by making sense of problems and persevering to solve them, using critical thinking skills to justify mathematical reasoning and connecting ideas to real world situations through modeling.

Content Standard(s):

IA.ACE.1* Create and solve equations and inequalities in one variable that model real-world problems involving linear, quadratic, simple rational, and exponential relationships. Interpret the solutions and determine whether they are reasonable.

IA.ACE.2* Create equations in two or more variables to represent relationships between quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales.

Jill owns an 1800 ft^2 house. She intends on installing hardwood floors throughout. Her house is a single story rectangular ranch style.

a. Give two possible dimensions that for this for the length and width of her house.
b. The warehouse sells hardwood by the square yard. How many square yards will be required to cover her entire floor?
c. Jill's house has two bathrooms. One has dimensions of 3ft x 6ft and the other 6ft x 6ft. If she decides to tile the floors in the bathroom instead of placing hardwood, how many yd^2 will she need of hardwood?
d. How many yd^2 of tile will she need to tile both bathroom floors?
e. If the cost of the hardwood flooring is \$3.00/yd^2 and the tile is \$4.00/yd^2, will Jill have enough money to cover both if her budget is \$300.
f. Justify your answer to show your thought process.

Connection of Standards:

Process Standard(s):

Students will show their understanding of **solving linear systems of equations** by making sense of problems, persevering, reasoning and making sense of relationships, using critical thinking skills to justify their mathematical reasoning, and connecting ideas to real world situations through modeling.

Content Standard(s):

IA.AREI.11* Solve an equation of the form $(x) = (x)$ graphically by identifying the $x$coordinate(s) of the point(s) of intersection of the graphs of $y = (x)$ and $y = (x)$.

You request a quote to rent a car for the weekend from two different companies. The Carousel Car Company will rent a car for a fee of \$75 and then an additional \$8/hr. The Road Runner Car Company will rent the same type of car for a fee of \$50 and then an additional \$10/hr. You're

not exactly sure if you want to rent the car for 12 hours or 24 hours, but would like to know which is the best price for either.

a. Which is the better deal for a 12 hour rental. Justify your answer.
b. Which is the better deal for a 24 hour rental. Justify your answer.
c. Is there a time when both rentals would be the same price? Justify your answer.

Think of something you would like to rent, a limo for the prom, a hall for a party a tuxedo for a wedding, anything of interest to you. Write two equations to represent two separate rental companies' charges. Answer the below questions for your scenario.

a. After how much time do the companies charge the same amount?
b. Justify your answer.

Connection of Standards:

Process Standard(s):

Students will show their understanding of **key features and transformations of quadratic functions given a visual representation** by making sense of problems and persevering to solve them, reasoning and making sense of relationships, and using critical thinking skills to justify mathematical reasoning.

Content Standard(s):

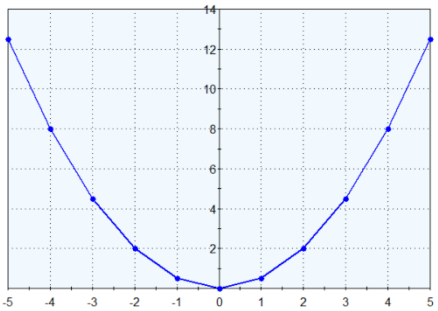
IA.ACE.1* Create and solve equations and inequalities in one variable that model real-world problems involving linear, quadratic, simple rational, and exponential relationships. Interpret the solutions and determine whether they are reasonable.

IA.ACE.2* Create equations in two or more variables to represent relationships between quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales.

IA.FBF.3* Describe the effect of the transformations (x), $(x)+k$, $f(x+k)$, and combinations of such transformations on the graph of $y = f(x)$ for any real number k. Find the value of k given the graphs and write the equation of a transformed parent function given its graph.

a. List as many key features for the below graph as you can identify. Include, but do not limit yourself to, intervals of increase and decrease, intercepts, and end behavior.

Graph $f(x)$



b. State the domain and range for this graph.
c. Is this graph vertically stretched or compressed from the parent function of $y = x^2$?
d. Write the equation for the graph. Justify how you derived this equation.

e. State the average rate of change over the interval [2,4].
f. Write a new equation for this graph if it is translated up 5 units.
g. Write a new equation for this graph if it is translated left 1 unit.
h. Write a new equation for this graph if it is reflected over the x-axis, translated up 5 units and left 1 unit.
i. Give the new domain and range for the transformed graph.

Connection of Standards:

Process Standard(s):

Students will show their understanding of **visual representations of quadratic functions, linear functions and exponential functions** by making sense of problems, persevering, reasoning and making sense of relationships, using critical thinking skills to justify their mathematical reasoning, and connecting ideas to real world situations through modeling.

Content Standard(s):

IA.FIF.9* Compare properties of two functions given in different representations such as algebraic, graphical, tabular, or verbal.

Below are three different equations written in functional notation.

a. Label the functions as linear, exponential or quadratic. How were you able to identify the different functions?
b. Evaluate each function for the domain {-2, 0, 3}.

$f(x) = 2x^2 - 4$ $g(x) = 5x + 10$ $h(x) = 3(2)^x$

_____ _____ _____

c. Can you predict which of the above functions will have a greater rate of change over the interval [-2,3]?
d. Without the parameters of a given domain, which of the three graphs do you predict would increase most rapidly as the domain moves towards infinity?
e. Justify your answer for d numerically.
f. Mathematically calculate the rate of change over the interval [-2,3]? Was your prediction correct?
g. Can you identify a real-world example to model each of the three equations? Explain each example.

Intermediate Algebra

Connection of Standards:
Process Standard(s):
Students will show their understanding of **writing and solving linear inequalities in two-variables** by making sense of problems, persevering, using critical thinking skills to justify their mathematical reasoning, and connecting ideas to real world situations through modeling.
Content Standard(s):
IA.ACE.1* Create and solve equations and inequalities in one variable that model real-world problems involving linear, quadratic, simple rational, and exponential relationships. Interpret the solutions and determine whether they are reasonable.

Tony is preparing for a marathon. He predicts he will need at least 635 ounces of water for the marathon. A small water bottle holds 16 ounces of water and a large bottle holds 20 ounces of water.

a. Write an inequality to represent this scenario.
b. How many small and large bottles will Tony require for the marathon?

Identify a relatable experience you have had that would require an inequality in two variables.

a. What makes this experience an inequality?
b. What are the two variables within the inequality?
c. Write an inequality to represent this experience.
d. Why is it important for you to know the values of the two variables?

Connection of Standards:
Process Standard(s): Students will show their understanding of **the complex number system and operations with imaginary numbers** by making sense of problems, persevering, reasoning and making sense of relationships, and using critical thinking skills to justify their mathematical reasoning.

Content Standard(s):
IA.NCNS.1* Know there is a complex number i such that $i2 = -1$, and every complex number has the form $a + bi$ with a and b real.

a. How does the complex number system vary from the real number system?
b. Why is the addition of imaginary numbers important when using higher-level mathematics?
c. Frank says that the difference between $3 + 2i$ and $6-2i$ is -3. Is he correct or incorrect? Justify your answer mathematically.
d. What do you wonder about imaginary numbers?

Intermediate Algebra

Connection of Standards:

Process Standard(s): Students will show their understanding of **identifying differences and commonalities amoung arithmetic and geometric sequences and modeling them through equations** by making sense of problems, persevering, reasoning and making sense of relationships, and using critical thinking skills to justify their mathematical reasoning.

Content Standard(s):

IA.FLQE.2* Create symbolic representations of linear and exponential functions, including arithmetic and geometric sequences, given graphs, verbal descriptions, and tables.

IA.FLQE.5* Interpret the parameters in a linear or exponential function in terms of the context.

The following sets of numbers are both sequences.

A. -1, 3, 7, 11, … B. -1, -4, -16, -64, …

a. How are these sequences the same?
b. How are these sequences different?

Think of the sequences in terms of a table of values, where the x-value is the number of the term and the sequence number is the corresponding y-value.

Example:

x	1	2	3	4
y	-1	3	7	11

c. If you graphed the table of values that corresponds with sequence A, what type of line would it create?
d. If you graphed the table of values that corresponds with sequence B, what type of line would it create?
e. Does this help you name the corresponding sequences as arithmetic or geometric? (Hint: an arithmetic sequence creates a linear graph and a geometric sequence creates an exponential graph.)
f. Explain in your own words how you can determine if a sequence is arithmetic or geometric.
g. Attempt to write an equation for both sequences.

Reflection:

Collaborate with someone in your family, a friend, or a neighbor. Ask them to look over your mathematical reasoning and ask you some guiding questions. Where were there struggles? Where did you triumph? What do you still wonder?

Now reflect on you work. Look over the content you covered, the processes that guided you through your discoveries, and think about your collaboration.

Write down your thoughts and allow your reflection to move you forward in your mathematical thinking.