

## Foundations in Algebra

The following resource will help to unify the concepts of Expressions, Functions and Inequalities by allowing students to investigate through analyzing graphs and creating personal representations of graphs. Students will also collaborate, create and reflect on their learning.

The organization of this resource connects process standards with content, allowing students the opportunity to connect their own ideas as they explore the mathematical content within the Foundations in Algebra Curriculum. It is also designed to allow multiple entry points to accommodate for different levels of understanding. Teachers have the flexibility of using the document in entirety or using specific standards to reinforce prior knowledge.

*(When students are prompted to justify their answers, they can show their reasoning in any way that makes sense to them. There are no incorrect means of justification as long as the student's thought process is visible.)*

---

### Connection of Standards:

#### Process Standard(s):

Students will show their understanding of **reading, interpreting and creating graphs** by making sense of problems, persevering, reasoning and making sense of relationships, using critical thinking skills to justify their mathematical reasoning, and connecting ideas to real world situations through modeling.

#### Content Standard(s):

FA.NQ.1\* Use units of measurement to guide the solution of multi-step tasks. Choose and interpret appropriate labels, units, and scales when constructing graphs and other data displays.

FA.NQ.2\* Label and define appropriate quantities in descriptive modeling contexts.

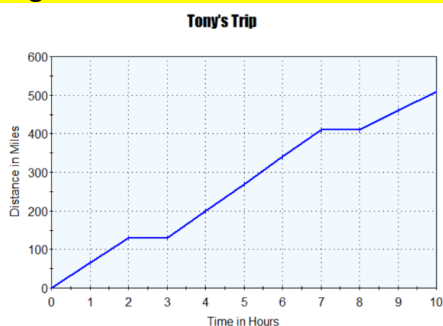
FA.NQ.3\* Choose a level of accuracy appropriate to limitations on measurement when reporting quantities in context.

FA.FIF.4\* Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features. Key features include intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity.

FA.AREI.10\* Explain that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.

Tony took a trip last month. Refer to the below graph to answer the questions about Tony's trip.

- How long did it take Tony to reach his final destination?
- Justify your answer. Why do you believe it took Tony that amount of time to finish his trip?
- How far do you believe Tony traveled?
- Justify your answer. Why do you believe Tony traveled that distance?
- When do you think Tony was driving the fastest?
- Why do you think he was driving the fastest during this time-period?
- How fast do you think Tony was driving, on average, for the 3 hours between hours 4 and 7?
- Did Tony ever stop?
- When do you think he stopped?
- What on the graph indicates that he stopped?



## Foundations in Algebra

Now it's your turn! Create a visual representation of an activity that you have done or plan to do. Label both axes with units and labels. After you are done with your visual representation, create 5 questions relating to your activity and answer the questions.

---

### Connection of Standards:

#### Process Standard(s):

Students will show their understanding of **reading, interpreting, analyzing and creating quadratic graphs** by making sense of problems, persevering, reasoning and making sense of relationships, using critical thinking skills to justify their mathematical reasoning, and connecting ideas to real world situations through modeling.

#### Content Standard(s):

FA.NQ.1\* Use units of measurement to guide the solution of multi-step tasks. Choose and interpret appropriate labels, units, and scales when constructing graphs and other data displays.

FA.NQ.2\* Label and define appropriate quantities in descriptive modeling contexts.

FA.NQ.3\* Choose a level of accuracy appropriate to limitations on measurement when reporting quantities in context.

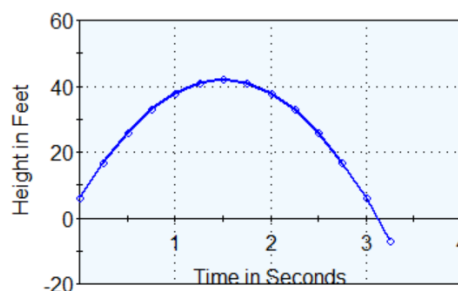
FA.FIF.4\* Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features. Key features include intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity.

FA.AREI.10\* Explain that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.

Barbara threw a ball into the air. Refer to the below graph to answer the questions about the ball's path.

- How long was the ball in the air?
- Justify your answer. Why do you believe the ball was in the air for that amount of time?
- How far away from Barbara was the ball when it hit the ground?
- Justify your answer. Why do you believe the ball was that distance from Barbara when it hit the ground?
- When do you think the ball was traveling at the greatest speed?
- Why do you think the ball was traveling the fastest during this time interval?
- When do you think the ball was traveling at its slowest speed?
- How does the graph justify this answer?
- Approximately, how high do you think the ball went in the air?
- How does the graph support this answer?

**Throwing a Ball**



## Foundations in Algebra

Create your own visual representation representing an activity that you have done in the past or that you plan to do that, if graphed, would create a parabolic curve (like the one shown above). Label both axes with units and labels. After you have created your visual representation, create 5 questions relating to the graph and answer the questions.

---

### Connection of Standards:

#### Process Standard(s):

Students will show their understanding of **reading, interpreting, analyzing and creating exponential graphs** by making sense of problems, persevering, reasoning and making sense of relationships, using critical thinking skills to justify their mathematical reasoning, and connecting ideas to real world situations through modeling.

#### Content Standard(s):

FA.NQ.1\* Use units of measurement to guide the solution of multi-step tasks. Choose and interpret appropriate labels, units, and scales when constructing graphs and other data displays.

FA.NQ.2\* Label and define appropriate quantities in descriptive modeling contexts.

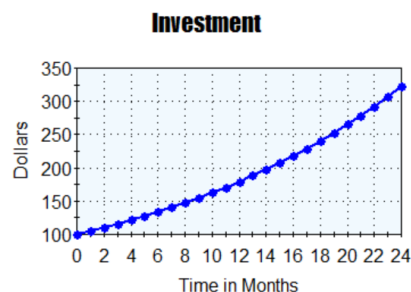
FA.NQ.3\* Choose a level of accuracy appropriate to limitations on measurement when reporting quantities in context.

FA.FIF.4\* Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features. Key features include intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity.

FA.AREI.10\* Explain that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.

Zachary wants to buy a new tv for his room that costs \$375. He decides to open an investment account that earns interest monthly. Refer to the below graph to answer the questions about the Zachary's investment account.

- How much money did Zachary deposit when he first opened the account?
- Justify your answer to the above question. How do you know that Zachary opened the account this amount of money?
- After 1 year, approximately how much money did Zachary have in his account?
- After 1 year, has Zachary saved at least half of the amount needed to purchase the tv?
- After how many months does Zachary have \$250 in his account?
- At the end of 2 years, does Zachary have enough money in his account to purchase the tv?
- If yes, after how many months did he have enough money in his account to purchase the tv? If no, predict how many months past the 2 years will Zachary have to wait before he has enough to purchase the tv.



## Foundations in Algebra

Think of a real world example that might represent exponential *decay* instead of growth. Draw a visual representation of this example. Label both axes with units and labels. After you have drawn the graph, create 5 questions relating to the graph and answer the questions.

---

### Connection of Standards:

#### Process Standard(s):

Students will show their understanding of the process of **rewriting radical expressions in rational exponent expression form and rewriting rational exponent expression form as radical expressions**. Understanding will be visible through the student's ability to justify the rule, by making sense of problems, reasoning and making sense of relationships and use critical thinking skills to justify mathematical reasoning.

#### Content Standard(s):

FA.NRNS.1\* Rewrite expressions involving simple radicals and rational exponents in different forms.

FA.NRNS.2\* Use the definition of the meaning of rational exponents to translate between rational exponent and radical forms.

$$\text{If } x^{\frac{p}{t}} = \sqrt[t]{x^p}, \text{ then } z^{\frac{1}{2}} = \sqrt{\quad}?$$

- Write a rule for the above statement of equality that shows the relationship between rational exponents and radicals.
- Justify your rule for equivalent expressions using all real numbers and no variables. Give at least 2 examples that shows your rule to be true.

---

### Connection of Standards:

#### Process Standard(s):

Students will show their understanding of **the real number system and properties of operations with real numbers** by reasoning and making sense of relationships and use critical thinking skills to justify mathematical reasoning.

#### Content Standard(s):

FA.NRNS.3 Explain why the sum or product of rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Samantha states that money is only represented using whole numbers. Is she correct? Justify your answer.

- Are all rational numbers integers? Justify your answer.
- Are all integers rational numbers? Justify your answer.
- Justify the following statements by giving two examples of each to show they are true.
- The sum of two rational numbers is rational.
- The product of two rational numbers is rational.
- The sum of a rational number and an irrational number is irrational.
- The product of a rational number and an irrational number is irrational.

## Foundations in Algebra

### Connection of Standards:

#### Process Standard(s):

Students will show their understanding of **modeling multi-step tasks and choosing accurate and appropriate measurement** by making sense of problems and persevering to solve them, using critical thinking skills to justify mathematical reasoning and connecting ideas to real world situations through modeling.

#### Content Standard(s):

FA.NQ.1\* Use units of measurement to guide the solution of multi-step tasks. Choose and interpret appropriate labels, units, and scales when constructing graphs and other data displays.

FA.NQ.2\* Label and define appropriate quantities in descriptive modeling contexts.

FA.NQ.3\* Choose a level of accuracy appropriate to limitations on measurement when reporting quantities in context.

Preston needs to install carpet in his  $1800 \text{ ft}^2$  house. He finds a carpet wholesaler that sells scrap left over pieces for a discounted price. The company tells Preston that they have a  $240 \text{ yd}^2$  piece to sell. Preston needs your help to determine if that is enough carpet to complete his job. (3 feet = 1 yard) Show your thinking by documenting your thought process as you try to answer this question.

---

### Connection of Standards:

#### Process Standard(s):

Students will show their understanding of **finding solutions to systems of linear equations** by making sense of problems, persevering, reasoning and making sense of relationships, using critical thinking skills to justify their mathematical reasoning, and connecting ideas to real world situations through modeling.

#### Content Standard(s):

FA.AREI.10\* Explain that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.

FA.AREI.6\* Solve systems of linear equations algebraically and graphically focusing on pairs of linear equations in two variables.

You request a quote to rent a boat for the weekend from two different companies. The Big Blue Boat Company will rent a boat for a fee of \$100 and then an additional \$5/hr. The Trellis Boat Company will rent the same type of boat for a fee of \$50 and then an additional \$10/hr. You're not exactly sure if you want to rent the boat for 6 hours or 12 hours, but would like to know which is the best price for either.

- Which is the better deal for a 6 hour rental. Justify your answer.
  - Which is the better deal for a 12 hour rental. Justify your answer.
  - Is there a time when both rentals would be the same price? Justify your answer.
-

## Foundations in Algebra

### Connection of Standards:

#### Process Standard(s):

Students will show their understanding of **transformations of functions and key characteristics of functions, including domain and range**, by making sense of problems and persevering to solve them, reasoning and making sense of relationships, and using critical thinking skills to justify mathematical reasoning.

#### Content Standard(s):

FA.FBF.3\* Describe the effect of the transformations  $(x)$ ,  $(x)+k$ ,  $f(x)+k$ , and combinations of such transformations on the graph of  $y=f(x)$  for any real number  $k$ . Find the value of  $k$  given the graphs and write the equation of a transformed parent function given its graph.

FA.FIF.5\* Relate the domain and range of a function to its graph and, where applicable, to the quantitative relationship it describes.

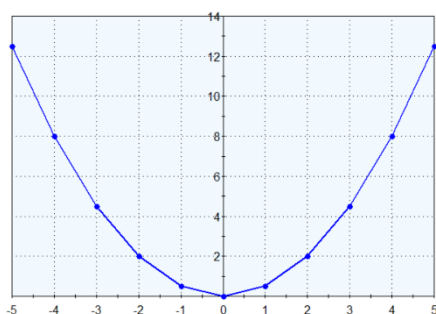
FA.FIF.7\* Graph functions from their symbolic representations. Indicate key features including intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity.

FA.FIF.9\* Compare properties of two functions given in different representations such as algebraic, graphical, tabular, or verbal. (Limit to linear; quadratic; exponential.)

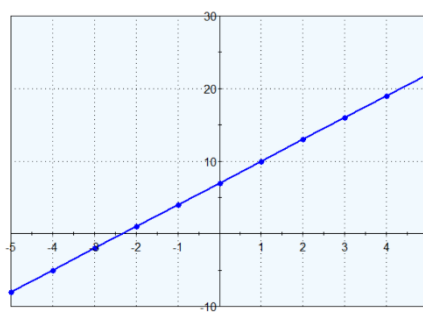
FA.FLQE.1\* Distinguish between situations that can be modeled with linear functions or exponential functions by recognizing situations in which one quantity changes at a constant rate per unit interval as opposed to those in which a quantity changes by a constant percent rate per unit interval.

Below you will find two graphs. Graph  $f(x)$  is quadratic and Graph  $g(x)$  is linear. Compare and contrast the graphs. List as many key characteristics of the two graphs as you can. Look at them closely. How are they different? Are there any characteristics that are the same? What are the intervals of increase and decrease? What are the intercepts? What is the end behavior?

Graph  
 $f(x)$



Graph  
 $g(x)$



State the transformation to  $f(x)$  if it were changed to  $f(x) + 5$ . (How would the graph of  $f(x)$  change?)

State the transformation to  $g(x)$  if it were changed to  $g(x - 2)$ . (How would the graph of  $g(x)$  change?)

## Foundations in Algebra

State the domain and range for  $f(x)$  and  $g(x)$ . How would the domain and range change after the transformations?

---

### Connection of Standards:

#### Process Standard(s):

Students will show their understanding of **linear, quadratic and exponential functions when given equations and investigating graphs through rate of change** and by making sense of problems, persevering, reasoning and making sense of relationships, using critical thinking skills to justify their mathematical reasoning, and connecting ideas to real world situations through modeling.

#### Content Standard(s):

FA.AREI.10\* Explain that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.

FA.FIF.2\* Evaluate functions and interpret the meaning of expressions involving function notation from a mathematical perspective and in terms of the context when the function describes a real-world situation

FA.FLQE.3\* Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or more generally as a polynomial function.

Below are three different equations written in functional notation.

- Label the functions as linear, exponential or quadratic. How were you able to identify the different functions?
- Evaluate each function for the domain  $\{-2, 1, 4\}$ .

$$f(x) = 3x^2 - 1$$

$$g(x) = 3x - 1$$

$$h(x) = 3(2)^x$$

- 
- Can you predict which of the above functions will have a greater rate of change over the interval  $[1, 4]$ ?
  - Without the parameters of a given domain, which of the three graphs do you predict would increase most rapidly as the domain moves towards infinity?
  - Justify your answer for d numerically.
  - Mathematically calculate the rate of change over the interval  $[1, 4]$ ? Was your prediction correct?
  - Can you identify a real-world example to model each of the three equations? Explain each example.

## Foundations in Algebra

---

### Connection of Standards:

#### Process Standard(s):

Students will show their understanding of **equations and inequalities as they apply to real world-problems** by reasoning to make sense of relationships and use critical thinking to justify mathematical reasoning.

#### Content Standard(s):

FA.ACE.1\* Create and solve equations and inequalities in one variable that model real-world problems involving linear, quadratic, simple rational, and exponential relationships. Interpret the solutions and determine whether they are reasonable

When would it be feasible to represent a real world problem with an inequality instead of an equation?

Suzy has 3 more pencils than Thomas.

- How would you represent this statement mathematically?
- Would an inequality represent it correctly? If yes, write the inequality. If no, explain why and write the correct expression?
- When words such as more or less are used within a statement, they can sometimes mean addition or subtraction or sometimes refer to an inequality. How might the wording within a statement be slightly different between the two?
- Write a statement for each example.

---

### Connection of Standards:

#### Process Standard(s):

Students will show their understanding of **analyzing solutions to linear inequalities in one-variable** by making sense of problems, persevering, using critical thinking skills to justify their mathematical reasoning, and connecting ideas to real world situations through modeling.

#### Content Standard(s):

FA.AREI.12\* Graph the solutions to a linear inequality in two variables.

Don takes his dog, Bruce, to the vet for a check-up. The veterinarian weighs Bruce and tells Don that he is overweight at 102 lbs. The vet tells Don that he would like Bruce to lose at least 2 lbs a week until he has reached his ideal weight of 84 lbs.

How long will it take for Don's dog to reach his ideal weight? Justify your answer.

---

### Connection of Standards:

#### Process Standard(s):



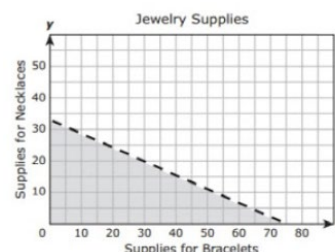
## Foundations in Algebra

Students will show their understanding of **graphing and analyzing linear inequalities in two-variables** by making sense of problems, persevering, reasoning and making sense of relationships, using critical thinking skills to justify their mathematical reasoning, and connecting ideas to real world situations through modeling.

**Content Standard(s):**

FA.AREI.12\* Graph the solutions to a linear inequality in two variables.

- Write a possible word problem for the given graph.
- Write an inequality to represent the word problem and graph.
- Give at least two justified solutions to the scenario.



---

### Connection of Standards:

**Process Standard(s):**

Students will show their understanding of how to **analyze equations written in different forms** by making sense of problems, persevering, reasoning and making sense of relationships, using critical thinking skills to justify their mathematical reasoning, and connecting ideas to real world situations through modeling.

**Content Standard(s):**

FA.FIF.8\* Translate between different but equivalent forms of a function equation to reveal and explain different properties of the function.

Andrew wrote the following equation  $3x - 2y = 0$ . He stated that the graph that represents the equation is linear and has a slope of 1.5.

- Is he correct in his assumption? Justify your answer by explaining or showing graphically why you think he is correct or incorrect.

---

### Reflection:

Collaborate with someone in your family, a friend, or a neighbor. Ask them to look over your mathematical reasoning and ask you some guiding questions. Where were there struggles? Where did you triumph? What do you still wonder?

Now reflect on your work. Look over the content you covered, the processes that guided you through your discoveries, and think about your collaboration.

Write down your thoughts and allow your reflection to move you forward in your mathematical thinking.