Algebra 2 Resource

Information Regarding This Resource The following resource will help to unify the concepts of Expressions, Functions and Inequalities and the Complex Number System by allowing students the opportunity to analyze data in a real-world scenario. Students will also collaborate, create and reflect on their learning.

The Format of This Resource This resource is organized into flexible components that can be utilized by educators, parents or students in its entirety or can be fragmented based on desired knowledge. Each text box contains the process skills students will use in the lesson to explore the mathematical content within the Algebra 2 Curriculum. The focus of each lesson is highlighted for easy reference. The lessons have been designed to allow multiple entry points to accommodate for different levels of understanding.

Throughout this resource, students are asked to justify or explain their answers, thought process or understanding. The intent is for students to reflect on their mathematical thoughts. Students should keep in mind that justifications or explanations can take multiple forms, including, but not limited to, diagrams, graphs, text, or pictures. These are not meant to be right or wrong, rather a means of making learning visible.

Connection of Standards:

Process Standard(s): Students will show their understanding of **performing operations with polynomial expressions** by making sense of problems, persevering, reasoning and making sense of relationships, and using critical thinking skills to justify their mathematical reasoning.

Content Standard(s):

A2.AAPR.1* Add, subtract, and multiply polynomials and understand that polynomials are closed under these operations.

Explain and justify using mathematical examples the resultant for the following polynomial operations:
a. The sum of two like degree polynomials…
b. The product of two like polynomials…

Create two additional mathematical examples to justify the following statements:
c. The difference of a cubic polynomial and a linear polynomial…
d. The product of two cubic polynomials…

For a polynomial to be closed under an operation, the resultant must be a polynomial.
e. using this definition, are polynomials closed under all 4 operations (+, -, x, ÷)?
f. If so, justify your answer with mathematical examples of all four operations. If not, justify your answer by giving a mathematical example that does not support the statement "polynomials are closed under all 4 operations".

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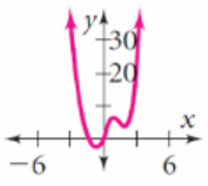
Connection of Standards:

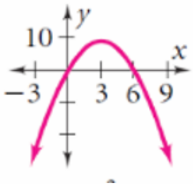
Process Standard(s): Students will show their understanding of **interpreting key features to identify different models of polynomial functions** by making sense of problems, persevering, reasoning and making sense of relationships, and using critical thinking skills to justify their mathematical reasoning.

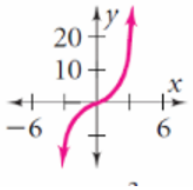
Content Standard(s):

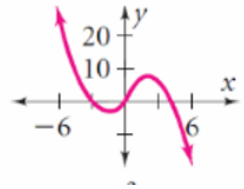
A2.FIF.4* Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features. Key features include intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity.

For the below graphs,
a. State the type of polynomial represented by each graph.
b. Identify the sign of the leading coefficient for each .
c. State the number of complex solutions each polynomial has, and
d. Identify the number of real and imaginary solutions for each.









Polynomial	_____	_____	_____	_____
Leading Sign	_____	_____	_____	_____
Complex Solutions	_____	_____	_____	_____
Real Solutions	_____	_____	_____	_____
Imaginary Solutions	_____	_____	_____	_____

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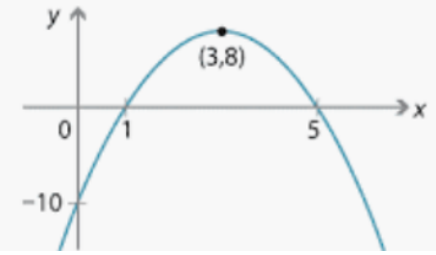
Connection of Standards:
Process Standard(s): Students will show their understanding of **writing quadratic equations given visual representations** by making sense of problems, persevering, reasoning and making sense of relationships, and using critical thinking skills to justify their mathematical reasoning.
Content Standard(s):
A2.ACE.2* Create equations in two or more variables to represent relationships between quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales.
A2.AAPR.3 Graph polynomials identifying zeros when suitable factorizations are available and indicating end behavior. Write a polynomial function of least degree corresponding to a given graph.

a. Write an equation for the graph.
b. Justify your reasoning for this equation.
c. How did the given points help in writing the equation?



Connection of Standards:
Process Standard(s): Students will show their understanding of **solving linear systems through linear programming** by making sense of problems, persevering, reasoning and making sense of relationships, using critical thinking skills to justify their mathematical reasoning, and connecting ideas to real world situations through modeling.
Content Standard(s):
A2.AREI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. Understand that such systems may have zero, one, two, or infinitely many solutions

We want to make pancakes and waffles. Pancakes require 3 cups of mix , 1 cup of milk, and 2 eggs. Waffles require 2 cups of mix, 2 cups of milk and 2 eggs. We have 24 cups of mix, 18 cups of milk, and 20 eggs available to make waffles and pancakes. The recipe for pancakes will make 5 servings and the recipe for waffles will make 6 servings. We want to optimize the number of each we can make.

a. Write the constraints for this problem.
b. Write the objective function for this problem.
c. Graph the constraints and identify the vertices of the feasible region.
d. Use the vertex points and the objective function to maximize the number of pancakes and waffles we can make.

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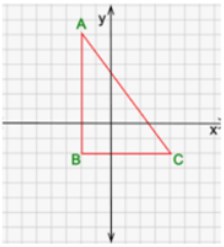
Connection of Standards:

Process Standard(s): Students will show their understanding of **solving literal equations** by making sense of problems, persevering, reasoning and making sense of relationships, and using critical thinking skills to justify their mathematical reasoning.

Content Standard(s):

A2.ACE.4* Solve literal equations and formulas for a specified variable including equations and formulas that arise in a variety of disciplines.

Revisit the Pythagorean Theorem formula from Geometry, $a^2 + b^2 = c^2$. In a right triangle positioned on a coordinate plane with both legs parallel to the x and y axes, a is the distance between the x-coordinates and b is the distance between the y-coordinates. (Refer to the picture.)



a. **Use this information to rewrite Pythagorean Theorem. Replace a and b, so that a = $(x - x_1)$ and b = $(y - y_1)$.**
b. **Now replace variable c with d and rewrite the formula in terms of d.**
c. **What do you notice about this new formula?**
d. **How does this information help you understand the distance formula?**
e. **Find the distance between points A and C.**
f. **Think of another formula you have used in another course, such as a science course, and rewrite that formula in terms of another variable.**
g. **Why is it important to be able to rewrite a formula in terms of another variable?**

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Connection of Standards:

Process Standard(s): Students will show their understanding of **writing and solving simple rational equations** by making sense of problems, persevering, reasoning and making sense of relationships, using critical thinking skills to justify their mathematical reasoning, and connecting ideas to real world situations through modeling.

Content Standard(s):

Solve simple rational and radical equations in one variable and understand how extraneous solutions may arise.

Sandy and Thomas are painting a room. Together, they paint the room in 5 hours. It normally takes Thomas 7 hours to paint the room alone. At this rate, how long would it take Sandy if she were painting alone?

The following equation can be written to solve this problem: $\frac{1}{s} + \frac{1}{7} = \frac{1}{5}$

a. **Explain why this equation makes mathematical sense.**
b. **How would the equation be different if Sandy's time working alone was given and the length of time it took both to paint the room was the unknown?**
c. **Estimate how long you think it would take Sandy to paint the room alone.**
d. **Mathematically solve for s and determine how close your estimation was.**
e. **Was your estimate close to the actual time? Does this make sense? Explain why it does or doesn't make sense.**
f. **Write your own work word problem and solve it.**

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Connection of Standards:

Process Standard(s): Students will show their understanding of **solving radical equations** by making sense of problems, persevering, reasoning and making sense of relationships, and using critical thinking skills to justify their mathematical reasoning.

Content Standard(s):

A2.AREI.2* Solve simple rational and radical equations in one variable and understand how extraneous solutions may arise.

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Justify the steps involved in solving the following radical equation.

$\sqrt[3]{(x-4)} = 2$ _____

$(x-4)^{\frac{1}{3}} = 2$ _____

$(x-4) = 2^3$ _____

$x - 4 = 8$ _____

$x = 12$ _____

a. **Check the solution to confirm it is not extraneous. Show all work.**
b. **Create a radical equation in one-variable and solve for that variable.**
c. **Justify each step of the process.**
d. **Check for extraneous solutions.**

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Connection of Standards:

Process Standard(s): Students will show their understanding of **identifying key features from a quadratic equation** by making sense of problems, persevering, reasoning and making sense of relationships, and using critical thinking skills to justify their mathematical reasoning.

Content Standards(s):

A2.AREI.4* Solve mathematical and real-world problems involving quadratic equations in one variable.

A2.FIF.4* Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features. Key features include intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity.

$$y = x^2 - 6x + 13$$

a. **Prior to solving the quadratic equation, make a prediction regarding the type of solutions the graph will have.**
b. **What was it about the equation that lead to your prediction?**
c. **Solve the equation using any method.**
d. **What method did you choose to solve the quadratic equation?**
e. **Why did you choose this method?**
f. **Was your original prediction correct?**
g. **Now that you know the solutions to the quadratic, can you predict the axis of symmetry?**
h. **Find the axis of symmetry.**
i. **Find the vertex.**
j. **Is the vertex a maximum or minimum?**
k. **State the intervals of increase and decrease.**
l. **State the end behavior.**
m. **State the domain and range.**
n. **Write an equation for a quadratic that has 2 real solutions.**
o. **Write an equation for a quadratic that have 1 real solution with multiplicity of 2.**
p. **Write an equation for a quadratic that has 2 imaginary solutions.**

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Connection of Standards:

Process Standard(s): Students will show their understanding of **writing and solving a system of equations (quadratic and linear)** by making sense of problems, persevering, reasoning and making sense of relationships, and using critical thinking skills to justify their mathematical reasoning.

Content Standards(s):

A2.AREI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. Understand that such systems may have zero, one, two, or infinitely many solutions.

$$y = x^2 + 5x - 10 \ \ and \ \ y = 3x - 2$$

a. **Given the following system of equations, how many solutions are POSSIBLE in this system? Justify your answer**
b. **Solve the system using any method.**
c. **How many solutions does this system have?**
d. **Write a system of quadratic equations that has exactly one solution. Justify the solution graphically or mathematically.**

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Connection of Standards:

Process Standard(s): Students will show their understanding of **interpreting key features of a quadratic model through an equation** by making sense of problems, persevering, reasoning and making sense of relationships, and using critical thinking skills to justify their mathematical reasoning.

Content Standards(s):

A2.AREI.4* Solve mathematical and real-world problems involving quadratic equations in one variable.

A2.FIF.4* Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features. Key features include intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity.

Use the following quadratic equation to answer a-i: $y = x^2 - 6x + 16.$

a. **Find the axis of symmetry**
b. **Find the vertex**
c. **Rewrite the above equation in Vertex Form.**
d. **Find the roots of the quadratic.**
e. **Which form, standard or vertex, did you use when solving for the roots?**
f. **Why did you choose this method? Are you happy with your choice or do you think it might have been more feasible to use the other method? Justify your answer.**
g. **Think about the different questions you can be asked about a quadratic model. What questions would be easiest answered with a quadratic written in standard form?**
h. **What questions would be easiest answered with a quadratic written in vertex form?**
i. **Explain your reasoning for g and h.**

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Connection of Standards:

Process Standard(s): Students will show their understanding of **identifying a sequence as arithmetic or geometric and writing an equation** by making sense of problems, persevering, reasoning and making sense of relationships, using critical thinking skills to justify their mathematical reasoning, and connecting ideas to real world situations through modeling.

Content Standards(s):

A2.FIF.3* Define functions recursively and recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

Francesca notices a pattern in a section of wallpaper. The pattern shows one bird in the first row, four birds in the second row, seven birds in the third row and so on. She states that the pattern represents an arithmetic sequence. She also states that the equation for the sequence is $A_n = 3n - 2$.

Francesca's friend, Nancy, is shocked that Francesca derived the equation so quickly! Do you have an idea as to how Francesca determined the equation without a pencil or paper?

Nancy also notices that the equation looks similar to one she learned in her Algebra class, y = mx + b. Nancy asks Francesca if there is a connection between the two equations.

Can you determine what Francesca's response will be to Nancy?

Is there a connection between an arithmetic sequence and the slope-intercept form equation?

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Connection of Standards:

Process Standard(s): Students will show their understanding of **writing an equation from transformations** by making sense of problems, persevering, reasoning and making sense of relationships, and using critical thinking skills to justify their mathematical reasoning.

Content Standards(s):

A2.FBF.3* Describe the effect of the transformations (x), $(x)+k$, $f(x+k)$, and combinations of such transformations on the graph of $y = f(x)$ for any real number k. Find the value of k given the graphs and write the equation of a transformed parent function given its graph.

$$y = 3(x+2)^2 - 1$$

Write a new equation for the above quadratic model that has been reflected over the x-axis, translated left 4 units and translated up 5 units.

Check at least 3 individual points to justify they have been reflected and translated correctly. You can justify mathematically or using a visual representation.

Connection of Standards:

Process Standard(s): Students will show their understanding of **identifying equivalent forms of imaginary numbers** by making sense of problems, persevering, reasoning and making sense of relationships, and using critical thinking skills to justify their mathematical reasoning.

Content Standards(s):

A2.NCNS.1* Know there is a complex number i such that $i2 = -1$, and every complex number has the form $a + bi$ with a and b real.

If $i = \sqrt{-1}$ and $i^2 = -1$, then what does $i^3 = ?$, what about $i^4 = ?$, $i^{26} = ?$

What pattern do you notice?

How could this pattern assist you in finding large degrees of i ?

Reflection:

a. **Collaborate with someone in your family, a friend, or a neighbor. Ask them to look over your mathematical reasoning and ask you at least 5 guiding questions. Document the 5 questions they asked.**
b. **Document the answers you gave to the 5 questions.**
c. **Reflect on your work. Where did you struggle? Where did you triumph? What do you still wonder?**
d. **Look over the content you covered, the processes that guided you through your discoveries, and think about your collaboration. Write down your thoughts and allow your reflection to move you forward in your mathematical thinking.**